

# HEALPix & rHEALPix Proj.4 Testing



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## Introduction

This document covers the testing procedures and results for the Proj.4 implementation of the HEALPix and rHEALPix projections. Testing conducted on 2012/06/07. Its difficult to construct a test plan that completely verifies that the projection implementations are correct. For both projections, points to be tested are selected based on their forward and inverse projections to be easily calculable by hand. Results from the Proj.4 implementation are to be compared by hand calculations and a Sage/Python implementation.

## Test Plan

A series of points are to be used for testing the projections. There are 3 categories of points: lon/lat, healpix, and rhealpix. The lon/lat category contains points represented as a latitude and longitude on a globe. The globe can either be a sphere or ellipsoid depending on the test case. The healpix category contains points that have been forward projected by the HEALPix projection. The rhealpix category contains points that have been forward projected by the rHEALPix projection.

## Test Case 1

The HEALPix forward projection is to be tested by using the spherical lon/lat category points by Proj.4's forward projection functions. The lon/lat points will be programmatically compared to the healpix category points. Any errors will be flagged and reported.

## Test Case 2

The HEALPix inverse projection is to be tested by using the healpix category points by Proj.4's inverse projection functions. The healpix points will be programmatically compared to the spherical lon/lat category points. Any errors will be flagged and reported.

## Test Case 3

The HEALPix forward projection is to be tested by using the WGS84 Ellipsoid lon/lat category points by Proj.4's forward projection functions. The lon/lat points will be programmatically compared to the healpix category points. Any errors will be flagged and reported.

## Test Case 4

The HEALPix inverse projection is to be tested by using the healpix category points by Proj.4's inverse projection functions. The healpix points will be programmatically compared to the WGS84 Ellipsoid lon/lat category points. Any errors will be flagged and reported.

## Test Case 5

The rHEALPix forward projection is to be tested by using the spherical lon/lat category points by Proj.4's forward projection functions. The lon/lat points will be programmatically compared to the rhealpix category points. Any errors will be flagged and reported. All possible polar region positions are tested. Both the northern and southern polar regions can have a position of 0 to 3 inclusive. This requires 16

experiments to fully test all polar regions.

## Test Case 6

The rHEALPix inverse projection is to be tested by using the rheapix category points by Proj.4's inverse projection functions. The rheapix points will be programmatically compared to the healpix lon/lat category points. Any errors will be flagged and reported.

## Test Case 7

The rHEALPix forward projection is to be tested by using the WGS84 Ellipsoid lon/lat category points by Proj.4's forward projection functions. The lon/lat points will be programmatically compared to the rheapix category points. Any errors will be flagged and reported.

## Test Case 8

The rHEALPix inverse projection is to be tested by using the rheapix category points by Proj.4's inverse projection functions. The rheapix points will be programmatically compared to the healpix lon/lat category points. Any errors will be flagged and reported.

## Definitions

In order for readability, a few mathematical functions have been defined as variables. These definitions are listed below.

Variable	Definition
$\phi$	$\text{asin}(2/3)$
sig	$\text{sqrt}(3 - 3*\text{sqrt}(3)/2)$
np0	$/4 * (1 - \text{sig})$
np1	$/4 * (2 - \text{sig})$
sp0	np0
sp1	$-1*\text{np1}$
nq	$(-1,0), (0,-1) \times (\text{np0},\text{np1}) + (-2, )$
sq	$(\text{nq0}, -1*\text{nq1})$
e	WGS48_E

## Test Points

The following points are to be used for testing and comparison.

Index	(sphere) lon/lat	(ellipsoid) lon/lat	healpix	rheapix
0	(0,0)	(0,auth_lat(0,e,True))	(0,0)	(0,0)
1	(0, $\phi$ )	(0,auth_lat( $\phi$ ,e,True))	(0, $\pi/4$ )	(0, $\pi/4$ )
2	(0,- $\phi$ )	(0,auth_lat(- $\phi$ ,e,True))	(0,- $\pi/4$ )	(0,- $\pi/4$ )
3	( $\pi/2$ ,0)	( $\pi/2$ ,auth_lat(0,e,True))	( $\pi/2$ ,0)	( $\pi/2$ ,0)
4	(- $\pi/2$ ,0)	(- $\pi/2$ ,auth_lat(0,e,True))	(- $\pi/2$ ,0)	(- $\pi/2$ ,0)
5	(- $\pi$ , $\pi/2$ )	(- $\pi$ ,auth_lat( $\pi/2$ ,e,True))	(-3* $\pi/4$ ,/2)	(-3* $\pi/4$ , $\pi/2$ )
6	(- $\pi$ ,- $\pi/2$ )	(- $\pi$ ,auth_lat(- $\pi/2$ ,e,True))	(-3* $\pi/4$ ,-/2)	(-3* $\pi/4$ ,- $\pi/2$ )

7	$(0, \pi/3)$	$(0, \text{auth\_lat}(\pi/3, e, \text{True}))$	$(\text{np0}, \text{np1})$	$(\text{nq0}, \text{nq1})$
8	$(0, -\pi/3)$	$(0, \text{auth\_lat}(-\pi/3, e, \text{True}))$	$(\text{sp0}, \text{sp1})$	$(\text{sq0}, \text{sq1})$

Where the `auth_lat` function calculates the authalic latitude of the given point with the defined eccentricity and if the inverse should be calculated.

## Results

Due to rounding errors from double and float computation, a variance of  $1e^{-8}$  is used for comparing the expected results and the produced results.

### Test Case 1

This tests the lon/lat to HEALPix projection otherwise known as the spherical HEALPix projection. There are no inconsistent coordinates. All coordinates have passed.

### Test Case 2

This tests the HEALPix to lon/lat projection otherwise known as the inverse spherical HEALPix projection. There are no inconsistent coordinates. All coordinates have passed.

### Test Case 3

This tests the WGS84\_E lon/lat to HEALPix projection otherwise known as the ellipsoidal HEALPix projection. The following table displays an inconsistency at the 7th decimal place.

Index	lon/lat	projection output	healpix reference
6	$(-\pi, \text{auth\_lat}(-\pi/2, e, \text{True}))$	$(-2.356194515018888, -1.5707963019683535)$	$(-2.356194490192345, -1.5707963267948966)$

### Test Case 4

This tests the HEALPix to WGS84\_E lon/lat projection otherwise known as the inverse ellipsoidal HEALPix projection. There are no inconsistent coordinates. All coordinates have passed.

### Test Case 5

This tests the lat/lon to rHEALPix projection otherwise known as the spherical rHEALPix projection. There are no inconsistent coordinates for all polar regions. All coordinates have passed.

### Test Case 6

This tests the rHEALPix to lon/lat projection otherwise known as the inverse spherical rHEALPix projection. There are no inconsistent coordinates. All coordinates have passed.

### Test Case 7

This tests the WGS84\_E lon/lat to rHEALPix projection otherwise known as the ellipsoidal rHEALPix projection. There are no inconsistent coordinates for all polar regions. All coordinates have passed.

### Test Case 8

This tests the rHEALPix to WGS84\_E lon/lat projection otherwise known as the inverse ellipsoidal rHEALPix projection. There are no inconsistent coordinates. All coordinates have passed.